QUANTUM ENTANGLEMENT AND QUANTUM INFORMATION

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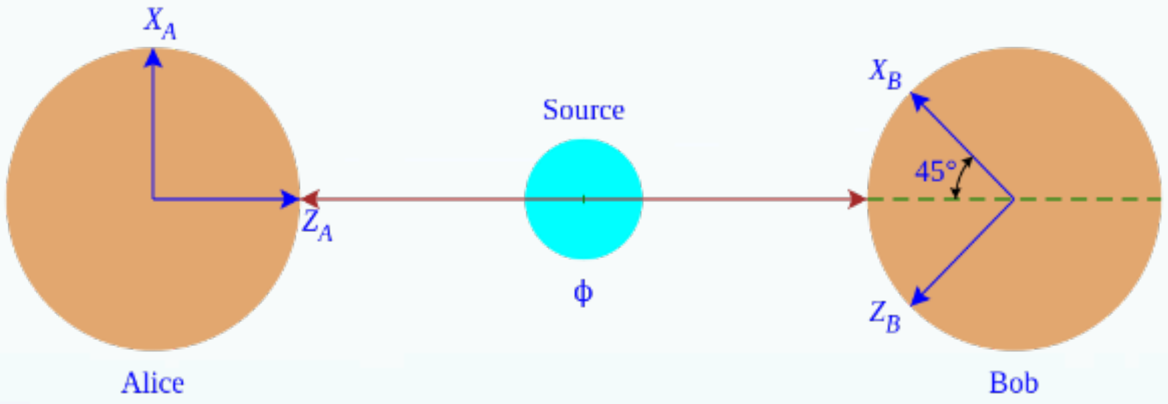
QUANTUM ENTANGLEMENT

- EPR (Einstein-Podolsky-Rosen) Paper 1934



QUANTUM ENTANGLEMENT

- EPR paper offered 2 explanations

1. Faster than light interaction (Einstein rejects this)
2. Hidden Variable

Hidden Variable theory states that there is a variable which has not been included QM yet, it contains all information.

- Bell's Theorem shows that 2nd argument is wrong.

There are four Bell pairs

QUANTUM ENTANGLEMENT

- **Bell Pairs**

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

$$|\Phi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B - |1\rangle_A \otimes |1\rangle_B)$$

$$|\Psi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |0\rangle_B)$$

$$|\Psi^-\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B - |1\rangle_A \otimes |0\rangle_B)$$

QUANTUM ENTANGLEMENT

- **Other typical entangled states**

1-GHZ (Greenberger-Horne-Zeilinger) state

$$|\mathrm{GHZ}\rangle = \frac{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}}{\sqrt{2}}.$$

2-W state

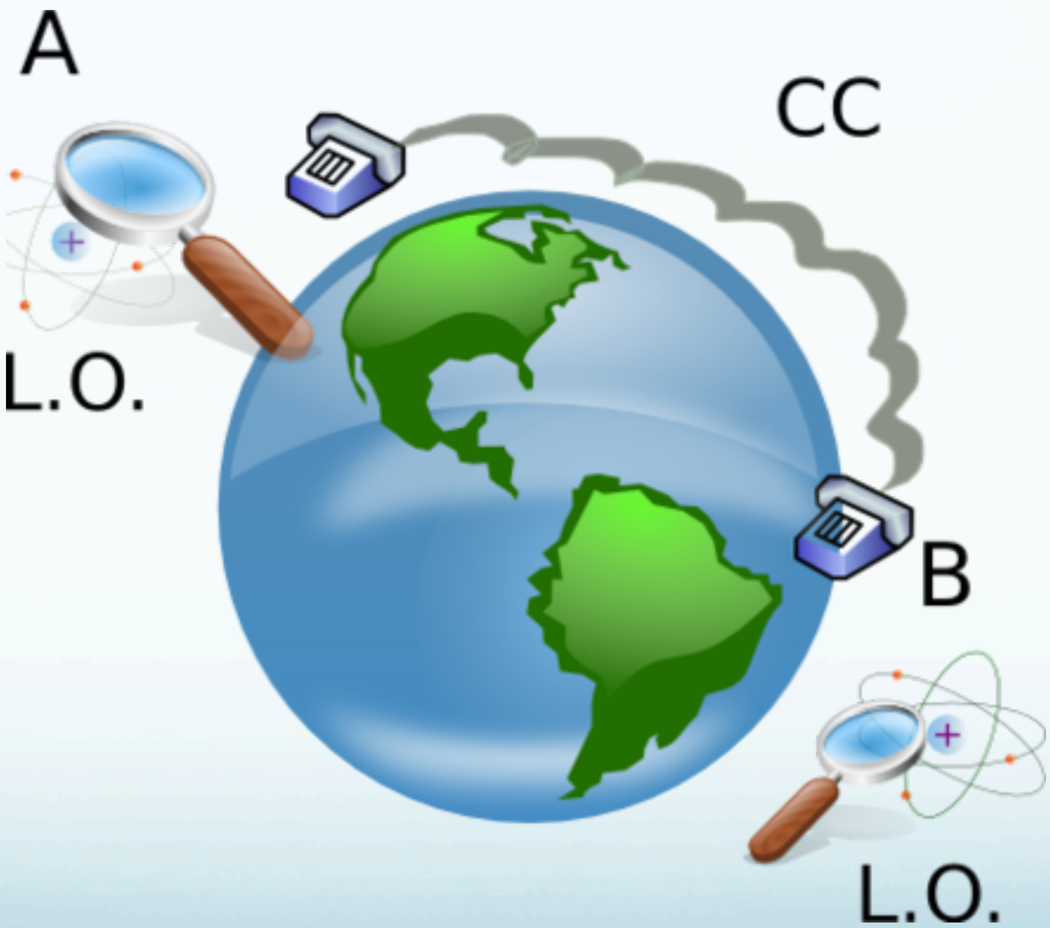
$$|W\rangle = \frac{1}{\sqrt{n}}(|100...0\rangle + |010...0\rangle + ... + |00...01\rangle)$$

3-NOON state

$$|\psi_{\mathrm{NOON}}\rangle = \frac{|N\rangle_a|0\rangle_b + |0\rangle_a|N\rangle_b}{\sqrt{2}},$$

QUANTUM ENTANGLEMENT

- Seperable states contains no entanglement
- All non-seperable states are entangled
- The entanglement of states does not increase under LOCC
- Entanglement does not change under local Unitary Operations
- There are maximally entangled states.


A
CC
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QUANTUM ENTANGLEMENT

- APPLICATION AREAS

1. Quantum Teleportation
2. Quantum Key Distribution
3. Quantum Computing

QUANTUM TELEPORTATION

- Quantum Teleportation is just a mechanism to "teleport" a qubit from one location to another one.

Alice and Bob are in different place and Alice wants to send an unkown qubit $|\psi\rangle_C = \alpha|0\rangle_C + \beta|1\rangle_C.$ to Bob.

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |0\rangle_B + |1\rangle_A \otimes |1\rangle_B)$$

From Bell states $|0\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$, etc.

$$|\Phi^+\rangle_{AB} \otimes |\psi\rangle_C =$$

$$\frac{1}{2}\Big[\ |\Phi^+\rangle_{AC} \otimes (\alpha|0\rangle_B + \beta|1\rangle_B)\ +\ |\Phi^-\rangle_{AC} \otimes (\alpha|0\rangle_B - \beta|1\rangle_B)$$

$$+\ |\Psi^+\rangle_{AC} \otimes (\beta|0\rangle_B + \alpha|1\rangle_B)\ +\ |\Psi^-\rangle_{AC} \otimes (\beta|0\rangle_B - \alpha|1\rangle_B)\Big].$$

QUANTUM TELEPORTATION

- Alice measures her state must be one of them

$$|\Phi^+\rangle_{AC}, |\Phi^-\rangle_{AC}, |\Psi^+\rangle_{AC}, |\Psi^-\rangle_{AC}$$

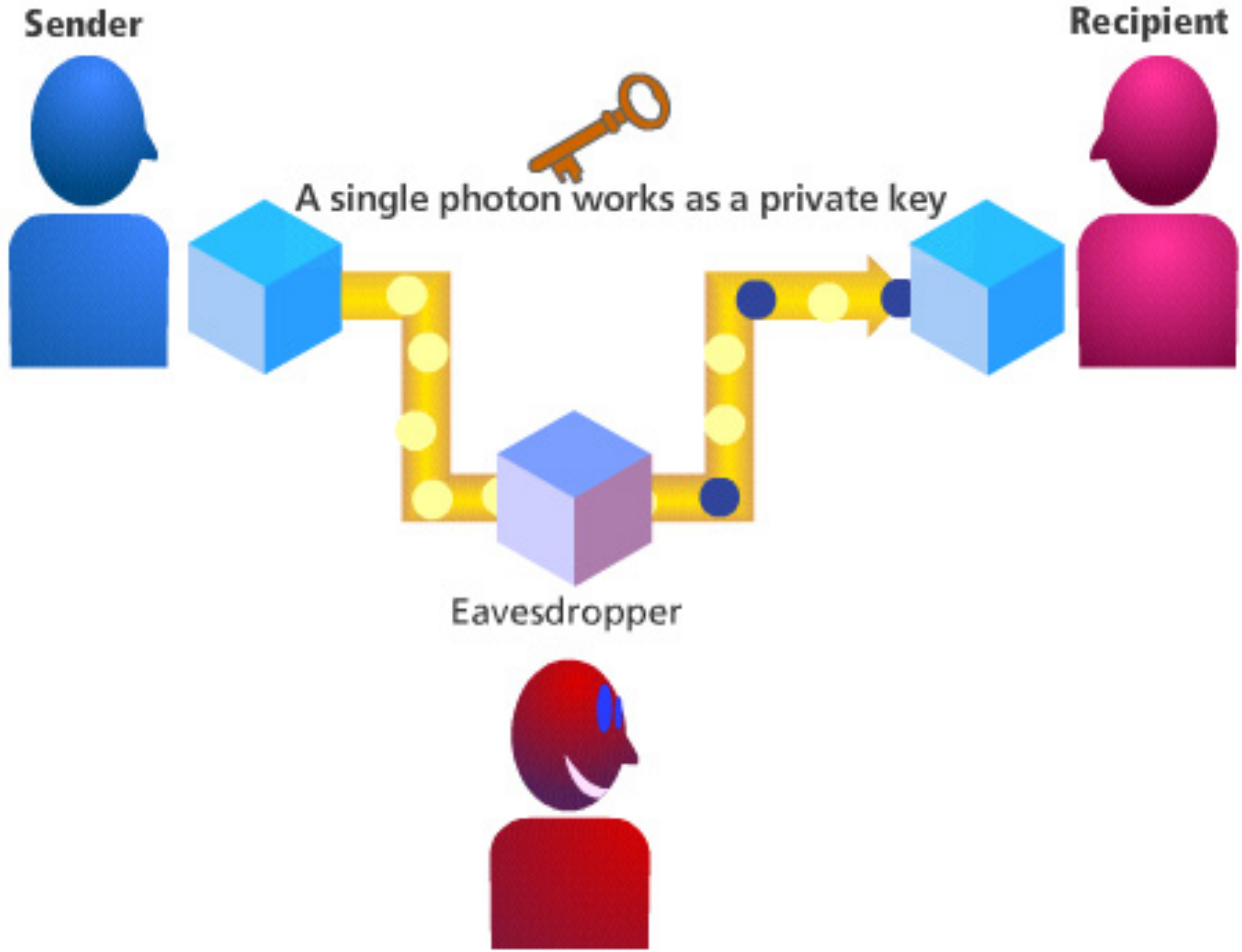
She send her state to Bob via classical channel. Then Bob decides operations on his state.

- If $|\Phi^+\rangle_{AC}$, Bob does not do anyting.
- If $|\Phi^-\rangle_{AC}$ Bob does an operation $\sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- If $|\Psi^+\rangle_{AC}$ Bob will do $\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
- For the remaining Bob does $-\sigma_3\sigma_1 = i\sigma_2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$

QUANTUM KEY DISTRIBUTION

QKD guarantees secure communication. It is based on entanglement.(E91 Protocol by Artur Ekert 1991)

1. Entanglement establishes exact synchronization.
2. Any attemp at eavesdropping destroys correlation and it is detected.



Recipients can discern the presence of eavesdroppers because

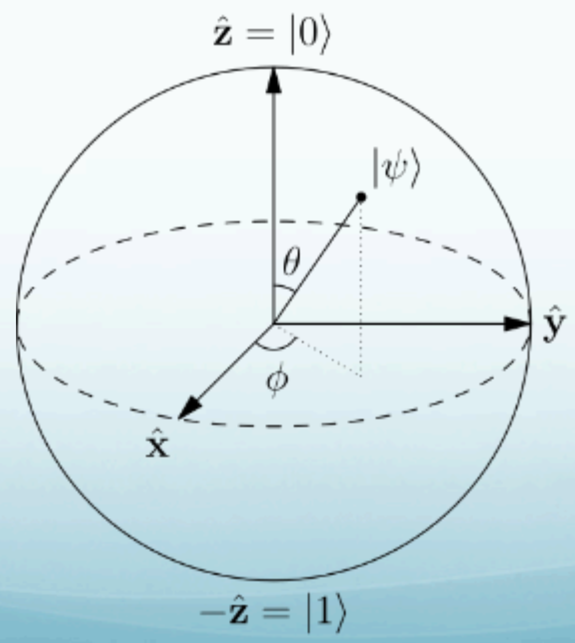
QUANTUM COMPUTING

- Quantum computing studies on quantum computers that make direct use superposition and entanglement to perform operations on data.

- Instead of bits quantum bits (qubits) are used.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

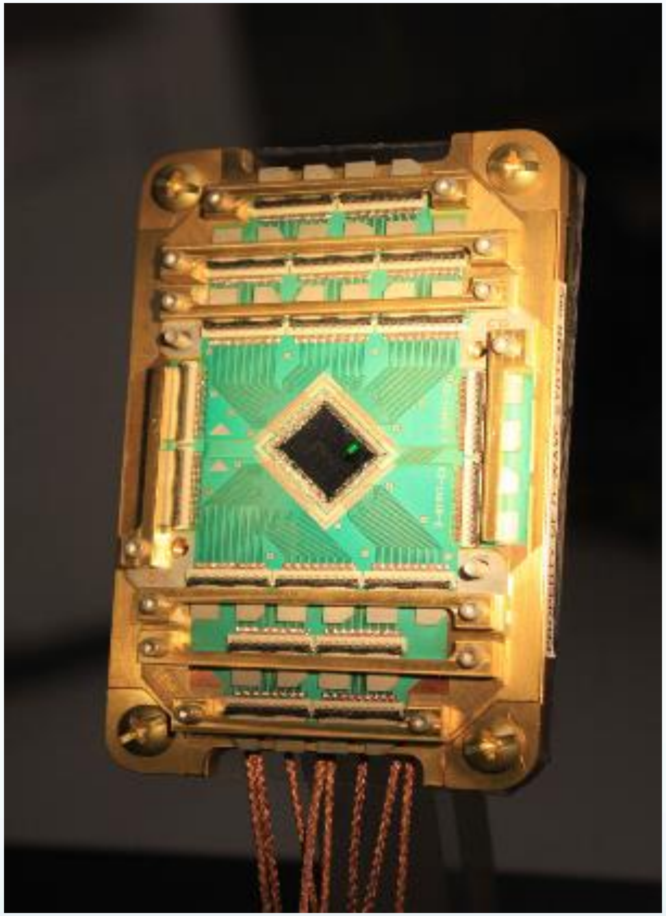
Bloch sphere is a representation of qubit.



QUANTUM COMPUTING

- N qubits are the superpostion of 2^N states
- Classical N bits just refers N states
- Quantum computers is used to simulate quantum many body systems

QUANTUM COMPUTING



512 qubit quantum computer developed by D-Wave.


SGI ICE X
SGI ICE X
sgi
sgi
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FINE TUNNING-q deformation

- q-deformation is used to fit theoretical solutions to experimental results in atomic level.
- Deformed boson algebra

$$a_q a_q^\dagger - q a_q^\dagger a_q = q^{-N},$$

$$[a_q, N_q] = a_q, \quad [a_q^\dagger, N_q] = -a_q^\dagger$$

$$[N]_q = a_q^\dagger a_q, \quad a_q a_q^\dagger = [N+1]_q.$$

$$a_q = \sqrt{\frac{q^N \psi_1 - q^{-N}\psi_2}{N(q - q^{-1})}}\,a,$$

$$a_q^\dagger = \sqrt{\frac{q^N \psi_1 - q^{-N}\psi_2}{N(q - q^{-1})}}\,a^\dagger,$$

$$N_q = N - (1/s) ln \psi_2.$$

FINE TUNNING-q deformation

- One can build deformed qubits and gates

Schwinger representation of angular momentum states

$$|jm\rangle = \frac{(a_1^\dagger)^{j+m}(a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|\tilde{0}_1\tilde{0}_2\rangle.$$

$$|x\rangle_q = (a_1^\dagger)_q^x (a_2^\dagger)_q^{(1-x)}|\tilde{0}_1\tilde{0}_2\rangle,$$

$$|1-x\rangle_q = (a_1^\dagger)_q^{1-x} (a_2^\dagger)_q^{(x)}|\tilde{0}_1\tilde{0}_2\rangle.$$

FINE TUNNING-q deformation

$$|x\rangle_q = \left(\sqrt{\frac{q^{N_1}\psi_1 - q^{-N_1}\psi_2}{N_1(q-q^{-1})}}a_1^\dagger\right)^x \left(\sqrt{\frac{q^{N_2}\psi_3 - q^{-N_2}\psi_4}{N_2(q-q^{-1})}}a_2^\dagger\right)^{1-x} |\tilde{0}_1\tilde{0}_2\rangle,$$

$$|1-x\rangle_q = \left(\sqrt{\frac{q^{N_1}\psi_1 - q^{-N_1}\psi_2}{N_1(q-q^{-1})}}a_1^\dagger\right)^{1-x} \left(\sqrt{\frac{q^{N_2}\psi_3 - q^{-N_2}\psi_4}{N_2(q-q^{-1})}}a_2^\dagger\right)^{x} |\tilde{0}_1\tilde{0}_2\rangle$$

FINE TUNNING-q deformation

$$\mathbf{Not_q} = \sum_{x=0}^{1} |1-x\rangle_q\, {}_q\langle x|,$$

$$\mathbf{Had_q} = -1^{(N_1)} + \sum_{x=0}^{1} |1-x\rangle_q\, {}_q\langle x|$$

$$\mathbf{Swap_q} = \sum_{x,y=0}^{1} |y\ x\rangle_q\, {}_q\langle y\ x|,$$

$$\mathbf{Cnot_q} = (1-N_1) + \sum_{x,y=0}^{1} |x\ 1-y\rangle_q\, {}_q\langle y\ x|N_1$$

$$\mathbf{Fredkin_q} = (1-N_1) + \sum_{x,y,z=0}^{1} |x\ z\ y\rangle_q\, {}_q\langle z\ y\ x|N_1,$$

$$\mathbf{Toffoli_q} = \sum_{x,y,z=0}^{1} |x\ y\ 1-z\rangle_q\, {}_q\langle z\ y\ x|\,[N_1 M_1 + (1-N_1)M_1] + \sum_{x,y,z=0}^{1} |x\ y\ 1-z\rangle_q\, {}_q\langle z\ y\ x|\,[(1-N_1)M_1 + (1-N_1)(1-M_1)]\,.$$

Constructing quantum logic gates using q-deformed harmonic oscillator algebras

ENTANGLEMENT MEASURES

1. CONCURRENCE
2. NEGATIVITY
3. RELATIVE ENTROPY OF ENTANGLEMENT
4. QUANTUM FISHER INFORMATION

CONCURRENCE

- Concurrence is an entanglement measure for a mixed state of two qubits

$$\mathcal{C}(\rho) \equiv \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4)$$

$\lambda_1, \ldots, \lambda_4$ are eigenvalues in descending order of a Hermitean matrix which is

$$R = \sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$$

Where

$$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$$

If C is 0 the state is seperable, and maximally entangled for 1.

NEGATIVITY

- $N(\rho)=\max\{0,-2\mu_{min}\}$

μ_{min} is the minimum eigenvalue of the partial transpose of the density matrix ρ

It can takes values between 0 and 1. If $N(\rho)$ is 0 the system is separable, and if it is 1 the system is maximally entangled as in concurrence.

RELATIVE ENTROPY OF ENTANGLEMENT

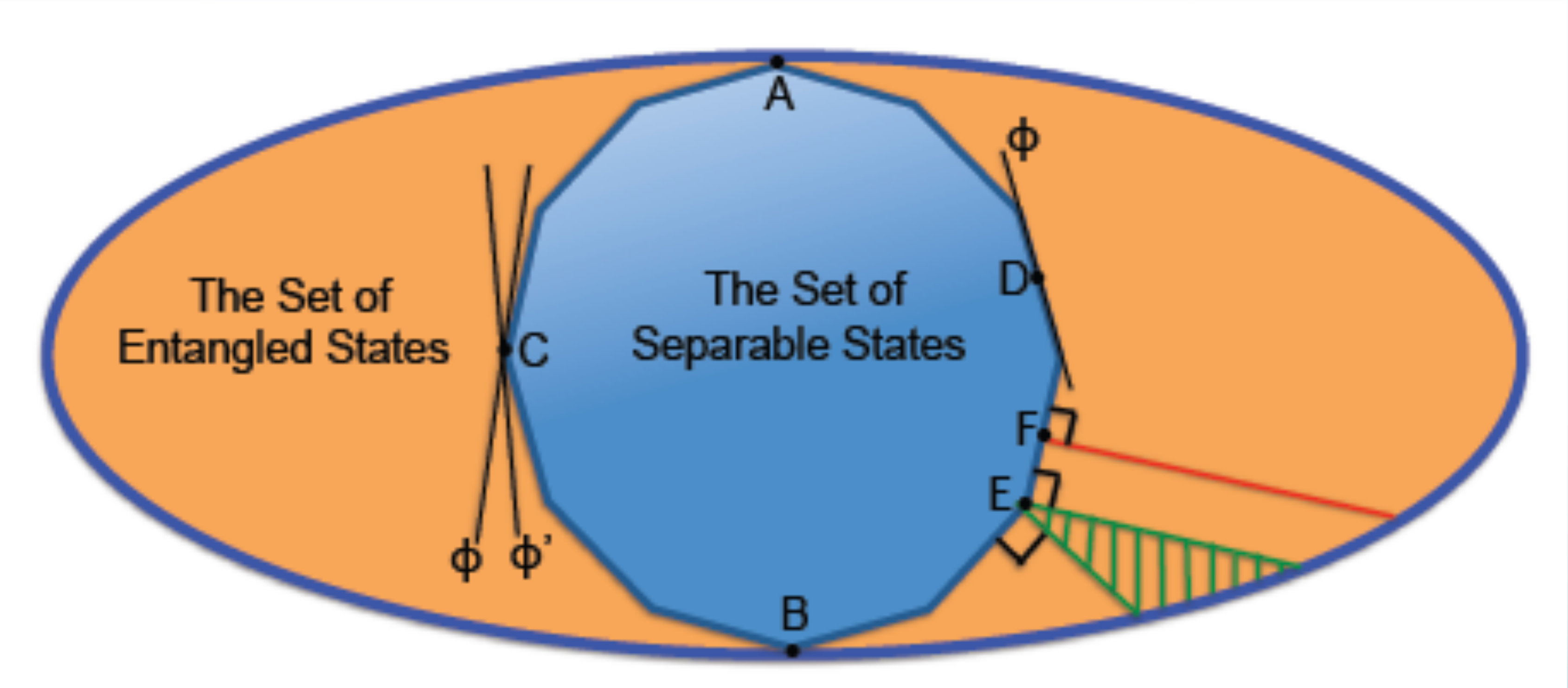
- $$E_R(\rho) = \min_{\sigma' \in \mathcal{D}} S(\rho||\sigma') = S(\rho||\sigma)$$

D is a set of separable states. S is quasidistance mesure

$$S(\rho||\sigma') = \mathrm{tr}\ (\rho \lg \rho - \rho \lg \sigma')$$

σ is a state on the boundary of separable states which is called closest separable state. If E is 0 the state is separable.

RELATIVE ENTROPY OF ENTANGLEMENT



QUANTUM FISHER INFORMATION

- Quantum Fisher Information (QFI) is related with estimating a parameter θ by making measurement on the operator A.
- How much precise can a phase be estimated.
- Cramér-Rao bound gives a lower bound on the pre

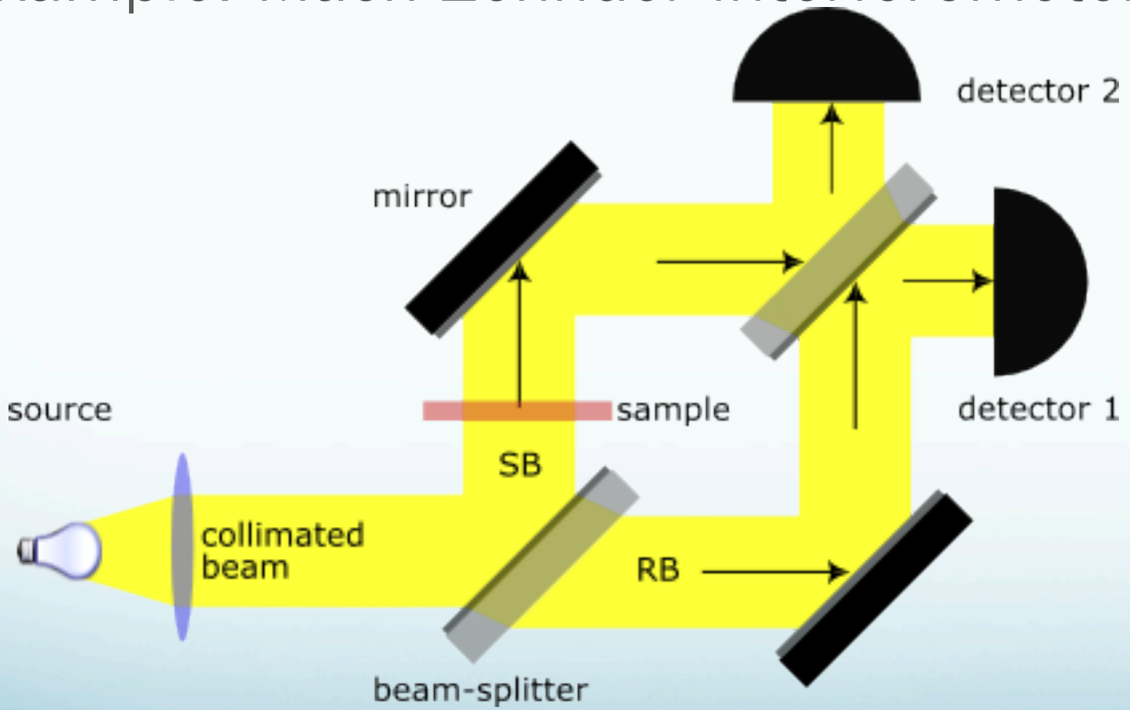
$$(\Delta\theta)^2 \geq \frac{1}{F_Q[\varrho, A]}$$

estimation

A is any Hermitean operator

QUANTUM FISHER INFORMATION

- Entanglement can increase the sensitivity of an interferometer.
- Example: Mach-Zehnder interferometer



QUANTUM FISHER INFORMATION

- For non entangled states $(\Delta\theta)^2 \sim \frac{1}{N}$ called as shot noise limit

- Quantum entanglement makes it possible to reach

$$(\Delta\theta)^2 \sim \frac{1}{N^2}$$

which is the Heisenberg limit

QUANTUM FISHER INFORMATION

- Quantum Fisher Information is defined as

$$F(\theta) = tr\rho(\theta)LL^{\dagger}$$

Quantum score has properties

$$\frac{\partial}{\partial\theta}\rho(\theta) = \rho(\theta)L \quad \text{right}$$

$$\frac{\partial}{\partial\theta}\rho(\theta) = L\,\rho(\theta) \quad \text{left}$$

$$\frac{\partial}{\partial\theta}\rho(\theta) = \frac{1}{2}\big(\rho(\theta)L + L\,\rho(\theta)\big) \quad \text{symmetric}$$

QUANTUM FISHER INFORMATION

From the general expression of QFI one can derive many formulas for QFI by using definitions of quantum score.

In our works we use

$$F(\rho, J_{\vec{n}}) = \sum_{i\neq j} \frac{2(p_i - p_j)^2}{p_i + p_j} |\langle i|J_{\vec{n}}|j\rangle|^2 = \vec{n}\, C\, \vec{n}^T.$$

$$C_{kl} = \sum_{i\neq j} \frac{(p_i - p_j)^2}{p_i + p_j} [\langle i|J_k|j\rangle\langle j|J_l|i\rangle + \langle i|J_l|j\rangle\langle j|J_k|i\rangle].$$

here p_i and |j> are the eigenvalue and eigenvector of state ρ

$$\overline{F}_{max} = \frac{F_{max}}{N} = \frac{\lambda_{max}}{N}$$

Quantum Fisher Information

- QFI for seperable states

$$\bar{F}_{max} \leq 1$$

For general states

$$\bar{F}_{max} \leq N$$

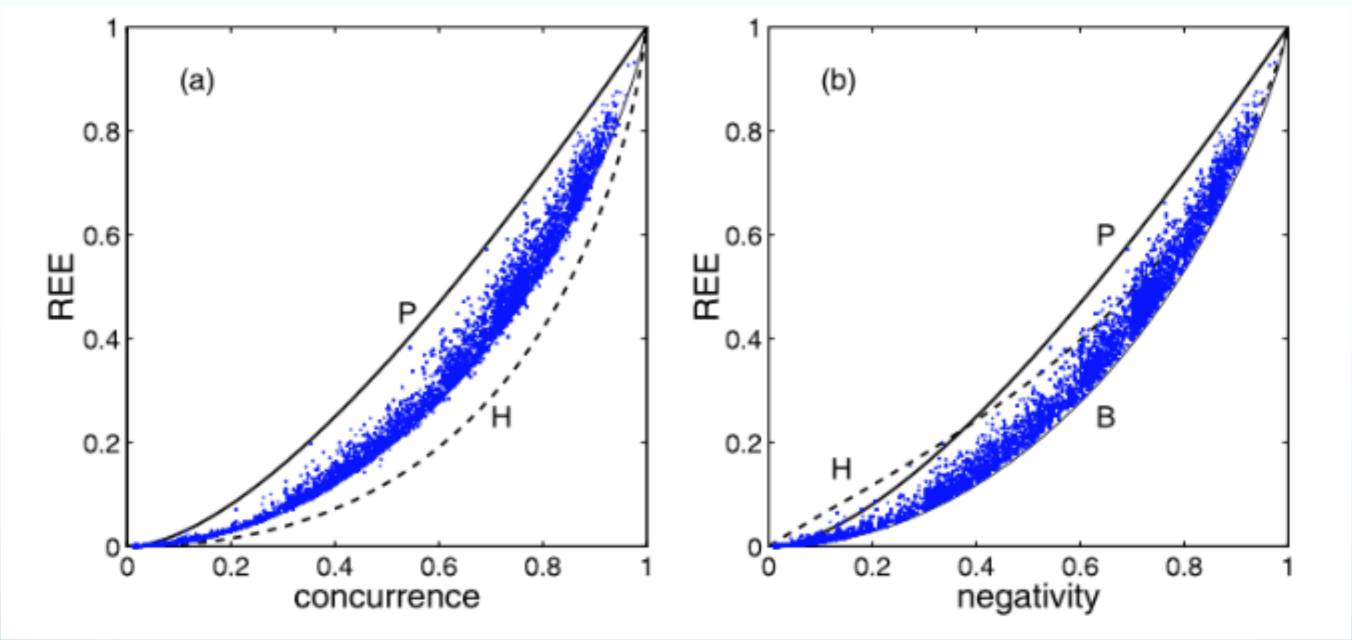
For maximally entangled states the upper limit is saturated.

QFI is not an entanglement measure!!

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS
J. Opt. B: Quantum Semiclass. Opt. 6 (2004) 542–548 PII: S1464-4266(04)86422-X

A comparative study of relative entropy of entanglement, concurrence and negativity

Adam Miranowicz and Andrzej Grudka



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Analysis of Entanglement Measures and LOCC Maximized Quantum Fisher Information of General Two Qubit Systems

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QUANTUM FISHER INFORMATION

Since QFI changes under unitary operations, one can find a unitary transformation in which QFI gives the same results with entanglement measures.

QUANTUM FISHER INFORMATION

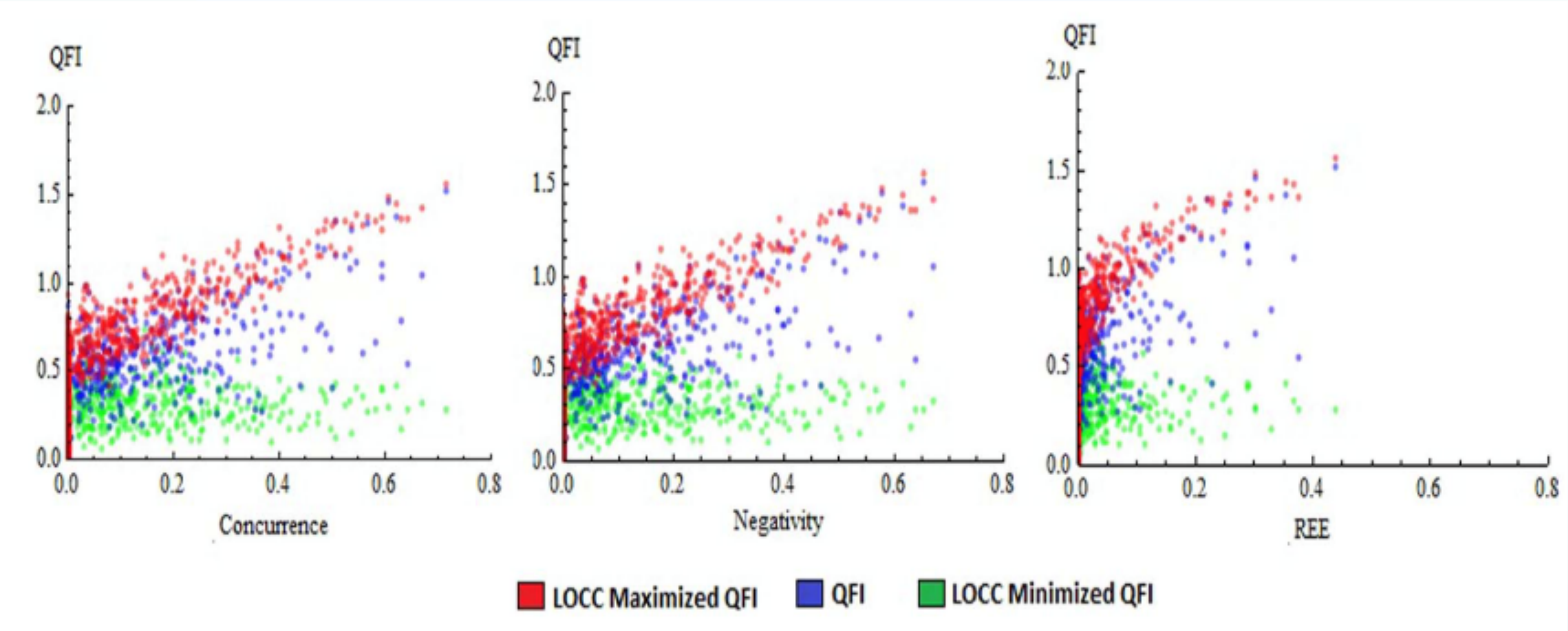
- An Euler rotation is defined on each qubit

$$U_{Rot}(\alpha,\beta,\gamma) = U_x(\alpha)U_z(\beta)U_x(\gamma)$$

Where the unitary rotations

$$U_j(\alpha) = \exp\left(-i\alpha\frac{\sigma_j}{2}\right), j \in \{x,z\}$$

QUANTUM FISHER INFORMATION



QUANTUM FISHER INFORMATION

- Maximal QFI is just the maximum eigenvalue of QFI matrix.
- Maximized QFI by LOCC behaves similar to the entanglement measures.
- If the rotation angle is chosen as smaller than 120 degree (which is used in this study) we can get better result.

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